

Non-vanishing U_{e3} under S_3 symmetry

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Abstract

This work proposes two models of neutrino masses that predict non-zero θ_{13} under the non-Abelian discrete flavor symmetry $S_3 \otimes \mathbb{Z}_2$. We advocate that the size of θ_{13} is understood as a group theoretical consequence rather than a perturbed effect from the tri-bi-maximal mixing. So, the difference of two models is designed only in terms of the flavor symmetry, by changing the charge assignment of righthanded neutrinos. The PMNS matrix in the first model is obtained from both mass matrices, charged leptons giving rise to non-zero θ_{13}^l and neutrino masses giving rise to tri-bi-maximal mixing. The physical mixing angles are expressed by a simple relation between θ_{13}^l and tri-bi-maximal angles to fit the recent experimental results. The other model generates PMNS matrix with non-zero θ_{13} , only from the neutrino mass transformation. The 5 dimensional effective theory of Majorana neutrinos obtained in this framework is tested with phenomenological bounds in the parametric spaces $\sin \theta_{23}, \sin \theta_{12}$ and m_2/m_3 vs. $\sin \theta_{13}$.

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I. INTRODUCTION

Recent long-baseline neutrino experiments, T2K [1] and MINOS [2], gave rise to the first indications to a non-zero U_{e3} . Following reactor neutrino experiments successfully presented certain values of $\sin^2 2\theta_{13}$. Double Chooz reported their first result $\sin^2(2\theta_{13}) = 0.085$ at the 68% CL [3]. Daya Bay narrowed down the range to $\sin^2(2\theta_{13}) = 0.092 \pm 0.005(\text{syst.})$ at 5.2σ [4], and RENO reported a definitive result with a value of $\sin^2(2\theta_{13}) = 0.113 \pm 0.019(\text{syst.})$ at 4.9σ [5]. The current bound on other angles, determined from neutrino oscillation experiments, are $0.490 \leq \sin \theta_{12} \leq 0.632$, and $0.583 \leq \sin \theta_{23} \leq 0.825$ at the 90% CL. The current data also include the mass squared differences that are accompanied by solar and atmospheric neutrino oscillations, $\Delta m_{sol}^2 \simeq (7_{-2}^{+10}) \times 10^{-5} eV^2$ and $\Delta m_{atm}^2 \simeq (2.5_{-0.9}^{+1.4}) \times 10^{-3} eV^2$, respectively [6, 7].

Non-Abelian discrete symmetries have provided theoretical frameworks for neutrino masses with tri-bi-maximal(TBM) mixing [8, 9] with $\sin \theta_{12} = 1/\sqrt{3}$, $\sin \theta_{23} = 1/\sqrt{2}$, and $\sin \theta_{13} = 0$ [10–18]. Due to the signals from recent measurements of θ_{13} , its non-zero value, which is still small relative to other two angles, is considered as being generated by a mechanism based on the symmetrical background rather than being a perturbation effect.

Two models with non-zero U_{e3} are introduced using $\mathbb{S}_3 \otimes \mathbb{Z}_2$ flavor symmetry. Both models have the same field contents with the same flavor charges, except for two righthanded neutrinos. Whether the \mathbb{S}_3 representations of the two fields are double **1**'s or a single **2**, the non-zero U_{e3} is obtained in charged lepton masses or in neutrino masses. Besides the Standard Model(SM) Higgs, a few scalar multiplets are added. The \mathbb{Z}_2 which commutes with \mathbb{S}_3 divides the fields by their parity, in the sense that all SM fields have even parity and so their couplings are not affected by the \mathbb{Z}_2 symmetry. The \mathbb{Z}_2 -odd righthanded neutrinos couple with only \mathbb{Z}_2 -odd scalar fields to make 5-dimensional Majorana masses in an effective theory. Using a simple assumption of Yukawa coupling constants, predictions of the mass ratios and mixing angles are presented.

This paper is organized as follows: Section II introduces the representations of flavor symmetry \mathbb{S}_3 , and contains the construction of Yukawa interactions of SM charged leptons with \mathbb{Z}_2 -even Higgs contents. In Section III, two models with non-zero U_{e3} are presented. The first model obtains the PMNS angle by a simple relation of TBM angles and the mixing angle θ_l of charged leptons. In the second model, the transformation of neutrino mass

TABLE I: Group representation of SM particles

Rep.	$(\mathbf{1}', 1)_F$	$(\mathbf{2}, 1)_F$	$(\mathbf{1}, 1)_F$
$(\mathbf{2}, -1/2)_G$	l_e	$L_\alpha : (l_\mu, l_\tau)$	H
$(\mathbf{1}, -1)_G$	e_r	$R_\alpha : (\mu_r, \tau_r)$	\dots

matrix becomes the PMNS matrix to the leading order. The predictions are examined in the figures. The conclusion section contains a summary and mentions some exclusion regions as the prediction, and an appendix is attached to describe the interactions of Higgs fields and their vevs to make the potential minimum.

II. DISCRETE FLAVOR SYMMETRY \mathbb{S}_3 AND YUKAWA INTERACTION

The minimal non-Abelian discrete symmetry \mathbb{S}_3 is the group of the permutation of the three sides of an equilateral triangle. There are six elements of the group in three classes, and their irreducible representations are $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{2}$. Its character table is mentioned in many models [19–22].

The Clebsch-Gordon coefficients in the real representations are given by the following product rules [23, 24],

$$\mathbf{1}' \times \mathbf{1}' = \mathbf{1} : ab, \quad (1)$$

$$\mathbf{1}' \times \mathbf{2} = \mathbf{2} : \begin{pmatrix} ab_2 \\ -ab_1 \end{pmatrix} \quad (2)$$

$$\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}, \quad (3)$$

$$\mathbf{1} : (a_1 b_1 + a_2 b_2)$$

$$\mathbf{1}' : (a_1 b_2 - a_2 b_1)$$

$$\mathbf{2} : \begin{pmatrix} a_2 b_2 - a_1 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix},$$

Here, an Abelian discrete symmetry \mathbb{Z}_2 is also adopted, which is the parity that distinguishes extra particles from the SM contents. The SM fields are assigned to representations of $\mathbb{S}_3 \otimes \mathbb{Z}_2$ as listed in Table I, where the SU(2) representation and hypercharge of a field are denoted by the subscription ‘G’ of the gauge symmetry, and the \mathbb{S}_3 representation and \mathbb{Z}_2 charge of the field are denoted by the subscription ‘F’. Although the \mathbb{Z}_2 charges of fields

are presented in Table I, \mathbb{Z}_2 symmetry does not affect the interactions among \mathbb{Z}_2 -even SM fields. Then, the Lagrangian of Yukawa couplings of the charged leptons and the Higgs scalar doublet H is

$$-\mathcal{L}_{SM} = c_1 H \bar{e}_r l_e + c_2 H \bar{R}_\alpha L_\alpha, \quad (4)$$

where the $SU(2)$ fermion doublet l_e is a flavor singlet but l_μ and l_τ belong to a doublet such as $L_\alpha \equiv (l_\mu, l_\tau)$ under \mathbb{S}_3 . Also, the righthanded charged lepton singlet e_r is a flavor singlet while μ_r and τ_r belong to a doublet such as $R_\alpha \equiv (\mu_r, \tau_r)$. The Higgs scalar doublet H of SM is involved in the above interactions as a flavor singlet. The Higgs self potential is

$$V_H = m_H^2 H^\dagger H + \frac{1}{2} \eta (H^\dagger H)^2, \quad (5)$$

so that, after spontaneous $SU(2)$ symmetry breaking, three Dirac mass matrices of the charged leptons from the Yukawa couplings become

$$c_i \langle H \rangle \sim \begin{pmatrix} c_1 v & 0 & 0 \\ 0 & c_2 v & 0 \\ 0 & 0 & c_2 v \end{pmatrix}. \quad (6)$$

It is shown that for the flavor model we can build a basis where the matrix of charged lepton masses is diagonal and $m_2 = m_3$, if only SM fields contribute to generate the Dirac masses. It follows that the right mass hierarchy is obtained from additional Yukawa couplings with additional scalar fields beyond the SM.

There is an additional Higgs scalar particle that couple with SM leptons, which is represented by, under $\mathbb{S}_3 \otimes \mathbb{Z}_2$,

$$(\mathbf{2}, 1)_F : \Phi (\varphi_1, \varphi_2). \quad (7)$$

The interactions of only \mathbb{Z}_2 -even Higgs particles, H , and Φ , among themselves are

$$\begin{aligned} V_e(H, \Phi) = & V_H + m_\varphi^2 \Phi^\dagger \Phi + \frac{1}{2} \Lambda (\Phi^\dagger \Phi)_r^2 \\ & + \lambda (\Phi^\dagger \Phi)_1 (H^\dagger H)_1 + \lambda' (\Phi^\dagger H)_2 (H^\dagger \Phi)_2 \\ & + \lambda'' \{ (\Phi^\dagger H)_2^2 + h.c. \} \\ & + \kappa \{ (\Phi^\dagger \Phi)_2 (\Phi^\dagger H)_2 + h.c. \}, \end{aligned} \quad (8)$$

where the term $\frac{1}{2} \Lambda (\Phi^\dagger \Phi)_r^2$ include such three contributions as

$$\Lambda (\Phi^\dagger \Phi)^2 = \lambda_a (\Phi^\dagger \Phi)_1^2 + \lambda_b (\Phi^\dagger \Phi)_{1'}^2 + \lambda_c (\Phi^\dagger \Phi)_2^2, \quad (9)$$

since the product $\Phi^\dagger \Phi$ can be any of the following representations, $(\mathbf{1}, 1)$, $(\mathbf{1}', 1)$ or $(\mathbf{2}, 1)$ of $\mathbb{S}_3 \otimes \mathbb{Z}_2$. According to the processes in Appendix in order to make the potential minimum, the vevs are obtained such that $\langle H \rangle = \langle H^\dagger \rangle = v$, $\langle \varphi_1 \rangle = v_1$, and $\langle \varphi_2 \rangle = 0$.

If the masses of charged leptons were obtained by using the Yukawa couplings in Eq.(6), the muon and tau lepton could have the same mass, $m_\mu = m_\tau$. Here, we introduce an additional contribution to the masses derived from the Yukawa couplings with another Higgs Φ .

$$-\Delta\mathcal{L}_{SM} = c_3\Phi\bar{R}_\alpha L_\alpha + c_4\Phi\bar{e}_\tau L_\alpha + c_5\Phi\bar{R}_\alpha l_e, \quad (10)$$

while the Yukawa couplings of quarks are protected from the non-SM additional Higgs Φ , since the flavor symmetry is leptonic so that all quarks are \mathbb{S}_3 singlets and \mathbb{Z}_2 -even. The Dirac mass matrix of charged leptons derived from both Eq.(4) and Eq.(10) has the following form

$$M_l \sim \begin{pmatrix} c_1 v & 0 & c_4 v_1 \\ 0 & c_2 v - c_3 v_1 & 0 \\ c_5 v_1 & 0 & c_2 v + c_3 v_1 \end{pmatrix}. \quad (11)$$

The masses of leptons are obtained from $U_l M_l^\dagger M_l U_l^\dagger = \text{Diag}(m_e^2, m_\mu^2, m_\tau^2)$. The transformation matrix U_l is required for Pontecorvo-Maki-Nakagawa-Sakata(PMNS) matrix along with the transformation of neutrino masses U_ν such that $U_{PMNS} = U_l^\dagger U_\nu$. We denote the 1-3 block of the matrix $M_l^\dagger M_l$ by K such as

$$K \equiv \begin{pmatrix} |c_1|^2 v^2 + |c_5|^2 v_1^2 & c_1^* c_4 v v_1 + c_5^* v_1 (c_2 v + c_3 v_1) \\ c_v^* c_1 v v_1 + (c_2^* v + c_3^* v_1) c_5 v_1 & |c_2 v + c_3 v_1|^2 \end{pmatrix}, \quad (12)$$

which is plausible by the relation in terms of the masses and the mixing angle as in $K = R(\theta_l, \delta_l) \text{Diag}(m_e^2, m_\tau^2) R^\dagger(\theta_l, \delta_l)$, where the 1-3 block rotation $R_{13}(\theta_l, \delta_l)$ is given by

$$R(\theta_l, \delta_l) \equiv \begin{pmatrix} \cos \theta_l & \sin \theta_l e^{-i\delta_l} \\ -\sin \theta_l e^{i\delta_l} & \cos \theta_l \end{pmatrix}. \quad (13)$$

The elements of the matrix K in Eq.(12) are described by physical parameters,

$$\begin{aligned} K_{11} &= m_e^2 \cos^2 \theta_l + m_\tau^2 \sin^2 \theta_l, \\ K_{22} &= m_\tau^2 \cos^2 \theta_l + m_e^2 \sin^2 \theta_l, \\ K_{12} &= K_{21}^* = (m_\tau^2 e^{i\delta_l} - m_e^2 e^{-i\delta_l}) \cos \theta_l \sin \theta_l. \end{aligned} \quad (14)$$

In opposite way, the mixing angle θ_l and the phase δ_l are obtained from the elements in Eq.(14) as

$$\tan 2\theta_l \cos \delta_l = \frac{K_{12} + K_{12}^*}{K_{22} - K_{11}}, \quad (15)$$

or from the matrix in Eq.(12)

$$\tan 2\theta_l = \frac{2\text{Re}[c_1^* c_4 v v_1 + c_5^* v_1 (c_2 v + c_3 v_1)]}{|c_2 v + c_3 v_1|^2 - |c_1|^2 v^2 - |c_5|^2 v_1^2}. \quad (16)$$

In general, the squared masses can be expressed in the following way,

$$\begin{aligned} m_e^2 &= \frac{1}{2} (K_{22} + K_{11}) - \frac{1}{2} (K_{22} - K_{11}) \sqrt{1 + \tan^2 2\theta_l \cos^2 \delta_l}, \\ m_\tau^2 &= \frac{1}{2} (K_{22} + K_{11}) + \frac{1}{2} (K_{22} - K_{11}) \sqrt{1 + \tan^2 2\theta_l \cos^2 \delta_l}, \end{aligned} \quad (17)$$

where $m_\mu^2 = |c_2 v - c_3 v_1|^2$. For $\tan^2 2\theta_l \ll 1$, the squared masses are approximated to

$$\begin{aligned} m_e^2 &\approx K_{11} = |c_1|^2 v^2 + |c_5|^2 v_1^2, \\ m_\tau^2 &\approx K_{22} = |c_2 v + c_3 v_1|^2, \end{aligned} \quad (18)$$

where m_e^2 and m_τ^2 are derived from the independent Yukawa couplings from each other, and M_l is close to a diagonal matrix due to the suppressed numerator in Eq.(16). For $\tan^2 2\theta_l \gg 1$, the squared masses in Eq.(17) reduce to

$$\begin{aligned} m_e^2 &= \frac{1}{2} (K_{22} + K_{11}) - \frac{1}{2} (K_{22} - K_{11}) \tan 2\theta_l \cos \delta_l, \\ m_\tau^2 &= \frac{1}{2} (K_{22} + K_{11}) + \frac{1}{2} (K_{22} - K_{11}) \tan 2\theta_l \cos \delta_l. \end{aligned} \quad (19)$$

The strong hierarchy between m_e^2 and m_τ^2 and the large mixing angle require a careful fine tuning in Eq.(19).

III. NEUTRINO MASSES UNDER \mathbb{S}_3 SYMMETRY

Two models are presented to explain neutrino masses in the normal hierarchy and the mixing matrix. Both models contain \mathbb{Z}_2 -odd additional particles, which can be distinguished from \mathbb{Z}_2 -even SM fields. The righthanded neutrinos are characterized by \mathbb{Z}_2 charge -1 and coupled with \mathbb{Z}_2 -odd Higgs fields, while the SM righthanded leptons are all assigned to the charge +1 and coupled with \mathbb{Z}_2 -even Higgs fields. Those additional Higgs contents are

$$\begin{aligned} (\mathbf{2}, -1)_F &: \Sigma (\sigma_1, \sigma_2) \\ (\mathbf{1}, -1)_F &: h. \end{aligned} \quad (20)$$

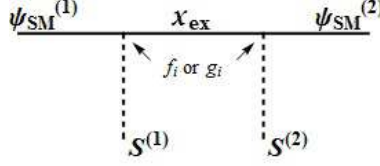


FIG. 1: Effective operator to give a mass to a Majorana particle. A 5-dimensional interaction of two scalars, $S^{(1)}$ and $S^{(2)}$, and two SM fermions, $\psi_{SM}^{(1)}$ and $\psi_{SM}^{(2)}$, is obtained by integrating out the heavy fermion x_{ex} in the internal line.

The Higgs potential which include the interactions of h and Σ is presented in Eq.(A4). Its minimum is obtained by the real vevs of the Higgs fields, which are denoted as $\langle h \rangle = \langle h^\dagger \rangle = u$ and $\langle \sigma_1 \rangle = \langle \sigma_2 \rangle = w$.

As far as the mixing of neutrino mass matrix is concerned, one model generates TBM, and the other generates nonzero θ_{13} in the PMNS matrix. Both types of mixing matrices are derived in terms of the \mathbb{S}_3 symmetry and its breaking mechanism by the vacuum expectation value(vev) of an \mathbb{S}_3 -doublet scalar field. The field contents and their charge assignments in the two models are identical to each other except the flavor charges of the righthanded neutrinos. The operators for Majorana masses have four external lines with an internal line, as shown in Fig.1. If the internal line is a heavy righthanded neutrino which has its ends coupled in a Yukawa interaction, the process giving rise to low energy effective masses is equivalent to the Seesaw Mechanism. Here, we describe the generation of neutrino masses while comparing the two models, the difference between which originated from the choice of group representations for the internal righthanded neutrinos.

A. Neutrino model for tri-bi-maximal U_ν

All additional fields beyond the SM, including righthanded neutrinos, are distinguished from the SM particles by \mathbb{Z}_2 parity. All the SM fields are \mathbb{Z}_2 even, so that the parity does not affect any interaction of SM particles. Additional Higgs scalars, $\Sigma \equiv (\sigma_1, \sigma_2)$ and h , and right-handed neutrinos, n_1, n_2, n_3 , all have \mathbb{Z}_2 -odd quantum number. Their representation under the gauge symmetry is $(\mathbf{1}, 0)_G$, and their representations under the flavor symmetry

are

$$\begin{aligned}(\mathbf{1}, -1)_F &: n_1 \\ (\mathbf{1}', -1)_F &: n_2, n_3.\end{aligned}$$

Their Yukawa interactions are as follows:

$$-\mathcal{L}_{ext} = f_0 h \bar{n}_1 l_e + f_1 \Sigma \bar{n}_1 L_\alpha + f_2 \Sigma \bar{n}_2 L_\alpha + f_3 \Sigma \bar{n}_3 L_\alpha, \quad (21)$$

where n_3 is redundant so that the same result can be obtained by just two righthanded neutrinos. But we keep the two identical singlets, n_2 and n_3 , for the comparison with other representation for them in another model. The couplings of Σ , n_i and L_α can be rephrased as $f_1 \Sigma \bar{n}_1 L_\alpha = f_1 \bar{n}_1 (\sigma_1 l_\mu + \sigma_2 l_\tau)$, $f_2 \Sigma \bar{n}_2 L_\alpha = f_2 \bar{n}_2 (\sigma_1 l_\tau - \sigma_2 l_\mu)$, and $f_3 \Sigma \bar{n}_3 L_\alpha = f_3 \bar{n}_3 (\sigma_1 l_\tau - \sigma_2 l_\mu)$. The gauge singlets n_i have Majorana masses as in $\frac{1}{2} M_i n_i n_i$, while the n_2 and n_3 have a cross term such as $M_x n_2 n_3$. If Majorana neutrinos n_i are very heavy, any two Yukawa couplings could be linked to each other as shown in Fig.1.

The vertex $f_i S^{(1)} \psi_{SM}^{(1)} \chi_{ex}$ or $f_i S^{(2)} \psi_{SM}^{(2)} \chi_{ex}$ in Figure 1 can correspond to any term in Eq.(21) if the n_i that is substituted into χ_{ex} is the same for both vertices. Then, an effective Lagrangian is obtained by integrating out the internal heavy fermion χ_{ex} .

$$\begin{aligned}-\mathcal{L}_{eff} &= \frac{f_0^2}{M_1} h h l_e l_e + \frac{f_0 f_1}{M_1} (h \sigma_1 l_e l_\mu + h \sigma_2 l_e l_\tau) \\ &+ \frac{f_1^2}{M_1} (\sigma_1 l_\mu + \sigma_2 l_\tau)^2 \\ &+ \left(\frac{f_2^2}{M_2} + \frac{f_3^2}{M_3} + 2 \frac{f_2 f_3}{M_x} \right) (\sigma_2 l_\mu - \sigma_1 l_\tau)^2.\end{aligned} \quad (22)$$

Here, M_i and M_x are the elements of mass matrix of singlet Majorana neutrinos n_i for $i = 1 - 3$. Since n_1, n_2 , and n_3 all belong to different representations, their Yukawa coupling constants f_i can be different and so can be their masses, M_i . When the scalar fields obtain vevs by spontaneous breaking of \mathbb{S}_3 symmetry, the above 5-dimensional interactions reduce to low-energy effective mass terms of light neutrinos $M_{ij}^{(\nu)} \nu_i \nu_j$. The symmetric matrix is

$$M^{(\nu)} = \frac{w^2}{M_1} \begin{pmatrix} f_0^2 x^2 & f_0 f_1 x & f_0 f_1 x \\ \sqrt{} & f_1^2 + \Delta f & f_1^2 - \Delta f \\ \sqrt{} & \sqrt{} & f_1^2 + \Delta f \end{pmatrix}, \quad (23)$$

and

$$\Delta f \equiv f_2^2 \varepsilon_2 + f_3^2 \varepsilon_3 + 2 f_2 f_3 \varepsilon_x, \quad (24)$$

where $x \equiv u/w$, $\varepsilon_2 \equiv M_1/M_2$, $\varepsilon_3 \equiv M_1/M_3$ and $\varepsilon_x \equiv M_1/M_x$. For simplicity, it is assumed that all f_i are 1; then the mass matrix in Eq.(23) has a simple pattern as follows:

$$\frac{w^2}{M_1} \begin{pmatrix} x^2 & x & x \\ \sqrt{\quad} & 1 + \Delta\varepsilon & 1 - \Delta\varepsilon \\ \sqrt{\quad} & \sqrt{\quad} & 1 + \Delta\varepsilon \end{pmatrix}, \quad (25)$$

where $\Delta\varepsilon \equiv \varepsilon_2 + \varepsilon_3 + 2\varepsilon_x$. The above matrix has a vanishing determinant, implying that one mass should be zero. The ratio of non-zero masses m_2/m_3 is $(2 + x^2)/2(\varepsilon_2 + \varepsilon_3 + 2\varepsilon_x)$. The type of mass hierarchy depends on the relative sizes of u and w and those of M_1 and $M_{2,3}$. If $x = 1$, furthermore, the matrix in Eq.(25) is exactly of the form that results from the tri-bi-maximal mixing: $U_{TBM} \cdot \text{Diag}(0, m_2, m_3) \cdot U_{TBM}^T$. Even when the Yukawa coupling constants, f_i , are allowed to be different from each other, θ_{13} and one of the masses vanish. Only the mass ratio m_2/m_3 is shifted to $(2f_1^2 + f_0^2 x^2)/2(f_2^2 \varepsilon_2 + f_3^2 \varepsilon_3 + 2f_2 f_3 \varepsilon_x)$. The specific pattern described above was discussed further in a previous work [18].

The PMNS matrix U_{PMNS} is obtained by

$$U_l^\dagger U_{TBM} = \begin{pmatrix} \cos \theta_l & 0 & -\sin \theta_l e^{i\delta_l} \\ 0 & 1 & 0 \\ \sin \theta_l e^{-i\delta_l} & 0 & \cos \theta_l \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (26)$$

where U_l is the transformation of M_l in Eq.(11) that imbeds $R(\theta_l, \delta_l)$ in Eq.(13) into the 1-3 block. On the other hand, if U_{PMNS} is expressed in the standard parametrization,

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \quad (27)$$

where c_{ij} and s_{ij} are $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively. From the comparison of two expressions in Eq.(26) and Eq.(27), the three angles in PMNS matrix are obtained by the following simple relations,

$$\sin \theta_{13} = \frac{1}{\sqrt{2}} \sin \theta_l \quad (28)$$

$$\sin \theta_{23} = \frac{1}{\sqrt{2 - \sin^2 \theta_l}} \quad (29)$$

$$\sin \theta_{12} = \frac{\cos \theta_l - \sin \theta_l}{\sqrt{3 - \frac{3}{2} \sin^2 \theta_l}}, \quad (30)$$

where $\delta_l = \pi$ in Eq.(26) and $\delta = 0$ in Eq.(27). The prediction of the model can be estimated with respect to the $\tan 2\theta_l$ in Eq.(16). The $\sin \theta_l$ in Eq.(28) matched to $0.292 < \tan 2\theta_l < 0.813$ can predict the range of θ_{13} measured in recent neutrino oscillation experiments [1–5]. Another predicted result is that θ_{23} belongs to the second octant. However, the range of θ_{12} predicted from the given θ_l is barely overlapped with the 3σ range of θ_{12} in the global analysis. A narrow range of $\tan 2\theta_l$, $0.292 < \tan 2\theta_l < 0.298$, can generate simultaneously the angles $0.10 < \sin \theta_{13} < 0.11$ and $0.51 < \sin \theta_{12} < 0.53$ marginally allowed in experiment, of which the eligibility is about to be tested by a higher precision data of the current oscillation experiments [1–5].

B. Neutrino model for a sizable θ_{13}

When the $\tan 2\theta_l$ is very small as considered in Eq.(18), i.e., when U_l is almost the unit, the PMNS matrix can be simply the transformation of neutrino mass matrix. The two righthanded neutrinos are in a doublet so they are not distinguishable in terms of flavor symmetry. The Majorana neutrinos n_2 and n_3 are tied into N_{II} , a two-dimensional representation of \mathbb{S}_3 , so that $N_{II} = (n_2, n_3)$. The scalar particles and n_1 do not change their charges, and all non-SM particles have \mathbb{Z}_2 -odd quantum number. The group representations of particles are summarized as follows:

$$\begin{aligned} (\mathbf{1}, -1)_F &: n_1 \\ (\mathbf{2}, -1)_F &: N_{II} (n_2, n_3). \end{aligned}$$

The Higgs potential is the same as the one for Model A in Eq.(A3). The Yukawa interactions are

$$\begin{aligned} -\mathcal{L}_{\text{ext}} &= g_0 \Sigma \bar{N}_{II} l_e + g_1 h \bar{N}_{II} L_\alpha + g_2 \Sigma \bar{n}_1 L_\alpha \\ &+ g_3 \Sigma \bar{N}_{II} L_\alpha. \end{aligned} \quad (31)$$

The couplings of Σ , h , n_i and L_α can be rephrased as $g_0 \Sigma \bar{N}_{II} l_e = g_0 (\bar{n}_2 \sigma_1 l_e + \bar{n}_3 \sigma_2 l_e)$, $g_1 h \bar{N}_{II} L_\alpha = g_1 (\bar{n}_2 h l_\mu + \bar{n}_3 h l_\tau)$, $g_2 \Sigma \bar{N}_{II} L_\alpha = g_2 \bar{n}_1 (\sigma_1 l_\mu + \sigma_2 l_\tau)$, and $g_3 \Sigma \bar{N}_{II} L_\alpha = g_3 \{ \bar{n}_2 (\sigma_2 l_\tau - \sigma_1 l_\mu) + \bar{n}_3 (\sigma_1 l_\tau + \sigma_2 l_\mu) \}$. The Majorana masses of the righthanded neutrinos are given by

$$\begin{aligned} &\frac{1}{2} M_1 n_1 n_1 + \frac{1}{2} M_2 N_{II} N_{II} \\ &= \frac{1}{2} M_1 n_1 n_1 + \frac{1}{2} M_2 (n_2 n_2 + n_3 n_3), \end{aligned} \quad (32)$$

where $N_{II}N_{II}$ becomes $n_2n_2 + n_3n_3$ according to the product rule in Eq.(3), and breaking of \mathbb{S}_3 gives rise to a common mass M_2 for n_2 and n_3 . For the same reason, the neutrinos have common Yukawa coupling constants g_0, g_1 , and g_3 . For very heavy Majorana neutrinos, any two Yukawa couplings can link to each other, as shown in Fig.1.

The interactions in Figure 1 can be considered as non-renormalizable 5 dimensional couplings obtained by integrating out the internal fermions n_i . Then the effective Lagrangian is given by

$$-\mathcal{L}_{eff} = \frac{g_0^2}{M_2}(\sigma_1\sigma_1 l_e l_e + \sigma_2\sigma_2 l_e l_e) + \frac{g_0 g_1}{M_2}(h\sigma_1 l_e l_\mu + h\sigma_2 l_e l_\tau) \quad (33)$$

$$+ \frac{g_0 g_3}{M_2}\{-\sigma_1\sigma_1 l_e l_\mu + \sigma_2\sigma_2 l_e l_\mu + 2\sigma_1\sigma_2 l_e l_\tau\} \quad (34)$$

$$+ \frac{g_2^2}{M_1}(\sigma_1 l_\mu + \sigma_2 l_\tau)^2 + \frac{g_1^2}{M_2}(h h l_\mu l_\mu + h h l_\tau l_\tau) \quad (35)$$

$$+ \frac{g_1 g_3}{M_2}\{-h\sigma_1 l_\mu l_\mu + h\sigma_2 l_\tau l_\tau + h\sigma_1 l_\mu l_\tau + h\sigma_2 l_\tau l_\mu\} \quad (36)$$

$$+ \frac{g_3^2}{M_2}\{(\sigma_2 l_\tau - \sigma_1 l_\mu)^2 + (\sigma_1 l_\tau + \sigma_2 l_\mu)^2\}, \quad (37)$$

where M_i are the heavy masses of singlet Majorana neutrinos n_i for $i = 1 - 3$. When the scalar fields obtain vevs by spontaneous breaking of \mathbb{S}_3 symmetry as shown in Eq.(A11)-Eq.(A12), the above 5-dimensional interactions reduce to low-energy effective mass terms of light neutrinos $M_{ij}^{(\nu)} \nu_i \nu_j$. The matrix is

$$M^{(\nu)} = \frac{w^2}{M_1} \begin{pmatrix} 2g_0^2\varepsilon & g_0 g_1 x\varepsilon & g_0 g_1 x\varepsilon + 2g_0 g_3\varepsilon \\ \sqrt{g_2^2 + g_1^2 x^2 \varepsilon - g_1 g_3 x\varepsilon + 2g_3^2 \varepsilon} & g_2^2 + g_1 g_3 x\varepsilon & \\ \sqrt{g_2^2 + g_1^2 x^2 \varepsilon + g_1 g_3 x\varepsilon + 2g_3^2 \varepsilon} & & \end{pmatrix}, \quad (38)$$

where $x \equiv u/w$, and $\varepsilon \equiv M_1/M_2$. For the simplest analysis, it is assumed that all g_i 's are one except g_0 . The g_0 is smaller than 1 to make m_1 smaller than m_2 .

A rather tedious estimation of masses and mixing angles from the above mass matrix is presented using pictorial analysis. Fig.2 presents the possible parametric plots that can arise from Eq.(38) in spaces of $\sin \theta_{13}$ vs. $\sin \theta_{23}$, $\sin \theta_{13}$ vs. $\sin \theta_{12}$, and $\sin \theta_{13}$ vs. m_2/m_3 , respectively. The value of g_0 is chosen as 0.7 in the following example. Five curves in each figure represent the choices of $x \equiv u/w$ as 0.0001, 0.1, 1.0, 2.0, and 5.0 by thick solid, dashed, dotted, dot-dashed, and thin solid lines, respectively. Four circles on each curve represent the choices of $\varepsilon \equiv M_1/M_2$ as 0.07, 0.17, 0.27, and 0.37 as $\sin \theta_{13}$ increases. The horizontal shadow in each

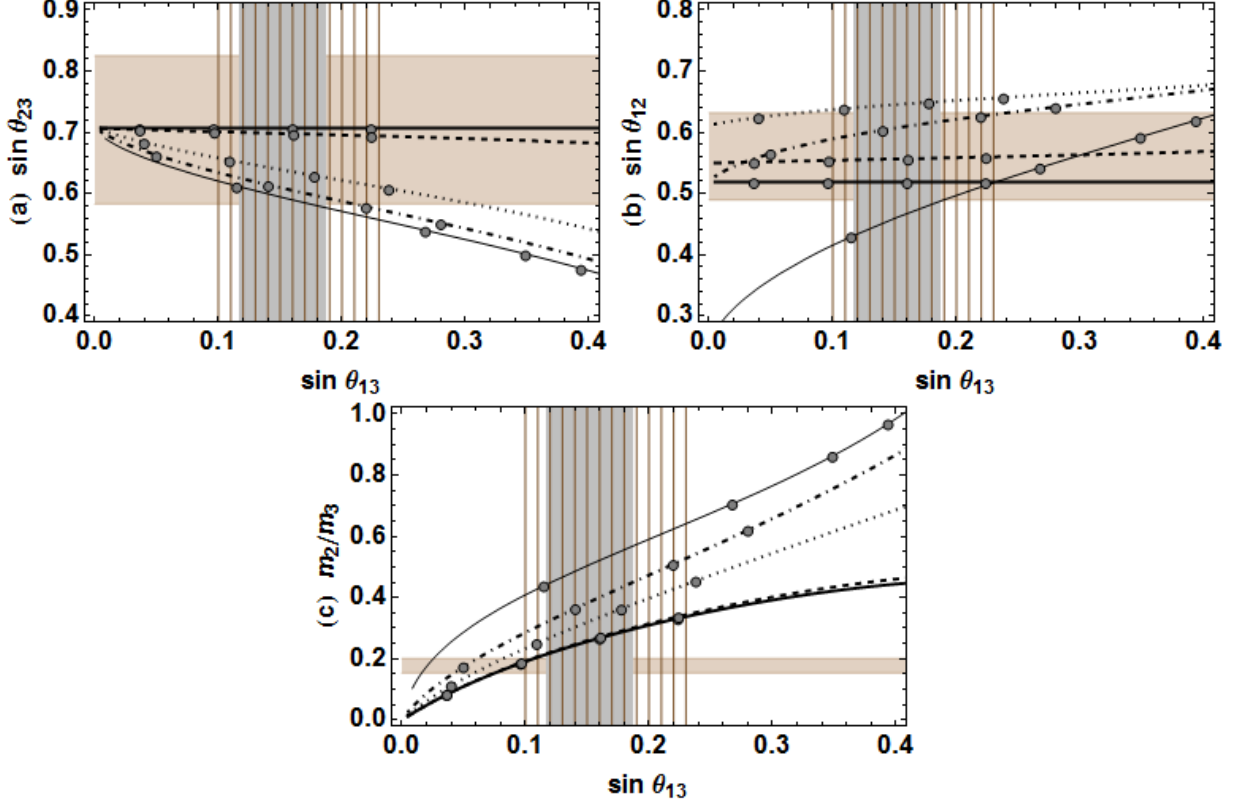


FIG. 2: Predictions by Model B. The brown shadow in each figure indicates the bound at the 90% CL for the angle and the mass ratio: $0.583 < \sin \theta_{23} < 0.825$, $0.490 < \sin \theta_{12} < 0.632$, and $0.154 < \sqrt{\Delta m_{21}^2 / \Delta m_{32}^2} < 0.201$. As for $\sin \theta_{13}$, the brown striped region indicates the RENO result: $0.100 < \sin \theta_{13} < 0.237$, and the gray shadowed region indicates Daya Bay result: $0.117 < \sin \theta_{13} < 0.187$. The thick solid, dashed, dotted, dot-dashed, and thin solid lines indicate $\langle h \rangle / \langle \Sigma \rangle \equiv x = 0.0001, 0.1, 1.0, 2.0$ and 5.0 in Eq.(38). The four small circles on each curve indicate $M_1/M_2 \equiv \varepsilon = 0.07, 0.17, 0.27$ and 0.37 in Eq.(38).

figure represents the allowed region at the 90% CL for the physical parameters according to the current experimental results: $0.583 < \sin \theta_{23} < 0.825$, $0.590 < \sin \theta_{12} < 0.632$, and $0.154 < \sqrt{\Delta m_{21}^2 / \Delta m_{32}^2} < 0.201$. As for the mass ratio, the region above 0.201 is not ruled out since m_2^2/m_3^2 can be larger than $\Delta m_{21}^2 / \Delta m_{32}^2$. For $\sin \theta_{13}$, two bounded regions are presented. One is $0.100 < \sin \theta_{13} < 0.237$ from RENO [5], which is indicated by brown stripes. The other is $0.117 < \sin \theta_{13} < 0.187$ from Daya Bay [4], which is indicated by gray shadows.

The bundle of curves in Figure 2 shows the area that the model can cover as M_1/M_2

increases. Different curves in a figure come from the relative ratio of the vevs of non-SM Higgs fields. The construction of mass matrix does not derive coefficients of elements, leaving them free parameters, when the symmetry builds up a pattern of a mass matrix based on the charge assignments of a flavor symmetry. Here, we examine the prediction from the model, while the effect of Yukawa couplings, f_i or g_i , is suppressed by setting them one. Whatever the value of M_1/M_2 is and whatever the ratio $\langle h \rangle / \langle \Sigma \rangle$ is, there is some area in $\sin \theta_{23}$, which is larger than 0.707, excluded by the prediction.

As for m_2/m_3 , likewise, the mass type of degeneracy or quasi-degeneracy is ruled out. The thin solid line in each figure describes the fit for $\langle h \rangle / \langle \Sigma \rangle = 5$, and figures 2(b) and 2(c) show that the resulted curves miss the allowed range, if $\langle h \rangle / \langle \Sigma \rangle > 5$. So the mass ratio range is confined within $m_2/m_3 < 0.6$ at most.

On the other hand, the ranges of M_1/M_2 and $\langle h \rangle / \langle \Sigma \rangle$ are also trimmed off by the experimental bounds of mixing angles. For instance, if $\langle h \rangle / \langle \Sigma \rangle < 0.1$, any value of M_1/M_2 smaller than 0.17 and that larger than 0.40 are excluded by RENO bound on θ_{13} .

IV. CONCLUSION

The recent measurements of $\sin^2(2\theta_{13})$ motivate an idea which is that non-zero $\sin \theta_{13}$ is generated by a mechanism based on the symmetrical background rather than being a perturbation effect from TBM with $\sin \theta_{13} = 0$. Two lepton models were introduced in terms of $\mathbb{S}_3 \otimes \mathbb{Z}_2$ flavor symmetry. One provides TBM from U_ν and non-zero θ_{13} from U_l to the PMNS matrix, while the other provides all leading orders of mixing angles from the neutrino mixing matrix, U_ν . The difference between two models is caused by the only difference between the flavor charge assignments for righthanded neutrinos. Other group theoretical properties are all the same for both models.

The \mathbb{Z}_2 symmetry splits the field contents into the particle fields beyond the SM and the fields in the SM, whether the charge is -1 or +1. Since the SM fields are \mathbb{Z}_2 -even, the Yukawa couplings of the SM fermions are protected from the coupling with a \mathbb{Z}_2 -odd scalar field. On the other hand, a \mathbb{Z}_2 -odd righthanded neutrino makes a vertex with a \mathbb{Z}_2 -odd scalar field. If a righthanded neutrino as an internal line is integrated out and the effective 5-dimensional coupling is suppressed by the mass scale of the righthanded neutrino, the Majorana masses of lefthanded neutrinos become then light. The \mathbb{Z}_2 -even scalar Higgs fields, H and Φ , in

Yukawa couplings contribute to the masses of charged leptons, and \mathbb{Z}_2 -odd scalar Higgs fields, h and Σ , in effective 5-dimensional couplings contribute to the masses of neutrinos.

Depending on the flavor charges of n_2 and n_3 among three generations, the type of neutrino mixing was determined. When they belong to separate $(\mathbf{1}', -1)_F$ representations, the model gave rise to the exact $\sin \theta_{13}^\nu = 0$, as shown in Eq.(25). When they belong to a single $(\mathbf{2}, -1)_F$ representation, the model gave rise to $\sin \theta_{13}^\nu \neq 0$ unless $M_1/M_2 = 0$. The prediction of the model, neglecting the contributions from most g_i , was studied in Fig.2. The results obtained for various ranges of the relative ratio, $\frac{M_1}{M_2}$, of Majorana masses and for those of the relative scales of vevs of non-SM Higgs, $\frac{\langle h \rangle}{\langle \Sigma \rangle}$, rule out the area of $\sin \theta_{23}$ larger than 0.707 and the mass pattern of (quasi-) degeneracy. Thus, the survival of model B can be determined, depending on whether $\theta_{23} < \pi/4$ and whether the mass type is hierarchical.

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Appendix A: Higgs Potential

The contents of Higgs scalar particles and their representations under $\mathbb{S}_3 \otimes \mathbb{Z}_2$ are

$$\begin{aligned}
(\mathbf{1}, 1)_F & : H \\
(\mathbf{2}, 1)_F & : \Phi (\varphi_1, \varphi_2) \\
(\mathbf{1}, -1)_F & : h \\
(\mathbf{2}, -1)_F & : \Sigma (\sigma_1, \sigma_2),
\end{aligned} \tag{A1}$$

which commonly belong to $(\mathbf{2}, 1/2)_G$ under $SU(2) \times U(1)$ gauge group. The full invariant Higgs potential can be organized into three parts as follows:

$$V = V_e(H, \Phi) + V_o(h, \Sigma) + V_\chi(H, \Phi; h, \Sigma), \tag{A2}$$

where V_e and V_o are the interactions of only \mathbb{Z}_2 -even particles and those of only \mathbb{Z}_2 -odd particles, respectively, while V_χ is the cross interactions of \mathbb{Z}_2 -even and \mathbb{Z}_2 -odd particles. Each contribution to the potential V is given as;

$$V_e(H, \Phi) = m_H^2 H^\dagger H + \frac{1}{2} \eta (H^\dagger H)^2 + m_\varphi^2 \Phi^\dagger \Phi + \frac{1}{2} \Lambda (\Phi^\dagger \Phi)_r^2 \quad (\text{A3})$$

$$+ \lambda (\Phi^\dagger \Phi)_1 (H^\dagger H)_1 + \lambda' (\Phi^\dagger H)_2 (H^\dagger \Phi)_2 + \lambda'' \{ (\Phi^\dagger H)_2^2 + \text{h.c.} \} + \kappa \{ (\Phi^\dagger \Phi)_2 (\Phi^\dagger H)_2 + \text{h.c.} \},$$

$$V_o(h, \Sigma) = m_h^2 h^\dagger h + \frac{1}{2} \lambda_h (h^\dagger h)^2 + m_s^2 \Sigma^\dagger \Sigma + \frac{1}{2} \Lambda_s (\Sigma^\dagger \Sigma)_r^2 \quad (\text{A4})$$

$$+ \lambda_s (\Sigma^\dagger \Sigma)_1 (h^\dagger h)_1 + \lambda'_s (\Sigma^\dagger h)_2 (h^\dagger \Sigma)_2 + \lambda''_s \{ (\Sigma^\dagger h)_2^2 + \text{h.c.} \} + \kappa_s \{ (\Sigma^\dagger \Sigma)_2 (\Sigma^\dagger h)_2 + \text{h.c.} \}.$$

$$V_\chi(H, \Phi; h, \Sigma) = \chi (H^\dagger H)_1 (h^\dagger h)_1 + \chi' (H^\dagger h)_1 (h^\dagger H)_1 + \chi'' \{ (H^\dagger h)_1^2 + \text{h.c.} \} \quad (\text{A5})$$

$$+ \lambda_\chi (\Phi^\dagger \Phi)_1 (h^\dagger h)_1 + \lambda'_\chi (\Phi^\dagger h)_2 (h^\dagger \Phi)_2 + \lambda''_\chi \{ (\Phi^\dagger h)_2^2 + \text{h.c.} \}$$

$$+ \eta_\chi (\Sigma^\dagger \Sigma)_1 (H^\dagger H)_1 + \eta'_\chi (\Sigma^\dagger H)_2 (H^\dagger \Sigma)_2 + \eta''_\chi \{ (\Sigma^\dagger H)_2^2 + \text{h.c.} \}$$

$$+ \gamma \{ (H^\dagger h)_1 (\Sigma^\dagger \Phi)_1 + \text{h.c.} \} + \gamma' \{ (H^\dagger \Sigma)_2 (h^\dagger \Phi)_2 + \text{h.c.} \}$$

$$+ \Gamma_\chi (\Phi^\dagger \Phi)_r (\Sigma^\dagger \Sigma)_r + \Gamma'_\chi (\Phi^\dagger \Sigma)_r (\Sigma^\dagger \Phi)_r + \Gamma''_\chi \{ (\Phi^\dagger \Sigma)_r^2 + \text{h.c.} \}.$$

The subscript ‘1’ or ‘2’ in each term indicates that the product of two fields belongs to the representation **1** or **2** in \mathbb{S}_3 . Each term with a subscript ‘r’ consists of three types of products, **1**, **1’** and **2** representations as in Eq.(9).

According to the product rules in Eqs. (1) - (3), $(\Phi^\dagger \Phi)_1 = |\varphi_1|^2 + |\varphi_2|^2$, $(\Phi^\dagger \Phi)_{1'} = \varphi_1^* \varphi_2 - \varphi_2^* \varphi_1$, and $(\Phi^\dagger \Phi)_2 = (|\varphi_2|^2 - |\varphi_1|^2 \quad \varphi_1^* \varphi_2 + \varphi_2^* \varphi_1)^T$. The Higgs potential in Eq.(A3) can be rephrased in terms of component fields $\{\varphi_i, \varphi_i^\dagger\}$ with $i = 1$ and 2 , and $\{H, H^\dagger\}$:

$$V_e(H, H^\dagger, \varphi_i, \varphi_i^\dagger) = m_H^2 |H|^2 + \frac{1}{2} \eta |H|^4 \quad (\text{A6})$$

$$+ (m_\varphi^2 + (\lambda + \lambda') |H|^2) \sum_i |\varphi_i|^2 + \lambda'' \{ H^2 (\varphi_1^{*2} + \varphi_2^{*2}) + \text{h.c.} \}$$

$$+ \frac{1}{2} (\lambda_a + \lambda_c) \sum_i |\varphi_i|^4 + (\lambda_a + \lambda_b) |\varphi_1|^2 |\varphi_2|^2 + \frac{1}{2} (\lambda_c - \lambda_b) (\varphi_1^{*2} \varphi_2^2 + \varphi_2^{*2} \varphi_1^2)$$

$$+ \kappa \{ H (2 |\varphi_2|^2 \varphi_1^* + (\varphi_2^*)^2 \varphi_1 - |\varphi_1|^2 \varphi_1^*) + \text{h.c.} \}.$$

When the Higgs particles obtain their real vacuum expectation values such that $\langle H \rangle = \langle H^\dagger \rangle = v$, $\langle \varphi_1 \rangle = v_1$, and $\langle \varphi_2 \rangle = v_2$, the potential can be expressed as follows.

$$V_e(v, v_1, v_2) = m_H^2 v^2 + m_\varphi^2 (v_1^2 + v_2^2) + \frac{1}{2} \eta v^4 \quad (\text{A7})$$

$$+ \frac{1}{2} \Lambda_a (v_1^2 + v_2^2)^2 + \Lambda_b v^2 (v_1^2 + v_2^2) + 2\kappa v (3v_2^2 v_1 - v_1^3),$$

where $\Lambda_a = \lambda_a + \lambda_c$, and $\Lambda_b = \lambda + \lambda' + 2\lambda''$.

Following the same steps as in Eq.(A3) - Eq.(A7), the potentials, V_o and V_χ , in terms of vevs, $\langle h \rangle = u$ and $(\langle \sigma_1 \rangle, \langle \sigma_2 \rangle) = (w_1, w_2)$, can be expressed as follows:

$$\begin{aligned} V_o(u, w_1, w_2) = & m_h^2 u^2 + m_s^2 (w_1^2 + w_2^2) + \frac{1}{2} \lambda_h u^4 \\ & + \frac{1}{2} \Lambda_s (w_1^2 + w_2^2)^2 + \Lambda_c u^2 (w_1^2 + w_2^2) + 2\kappa_s u (3w_2^2 w_1 - w_1^3), \end{aligned} \quad (\text{A8})$$

where $\Lambda_c \equiv \lambda_s + \lambda'_s + 2\lambda''_s$.

$$\begin{aligned} V_\chi = & k_1 u^2 v^2 + k_2 u^2 (v_1^2 + v_2^2) + k_3 v^2 (w_1^2 + w_2^2) + k_4 uv (v_1 w_1 + v_2 w_2) \\ & + k_5 v_1 v_2 w_1 w_2 + k'_5 (v_1^2 + v_2^2) (w_1^2 + w_2^2) + k''_5 (v_2^2 - v_1^2) (w_2^2 - w_1^2) + k'''_5 (v_1^2 w_1^2 + v_2^2 w_2^2), \end{aligned} \quad (\text{A9})$$

where $k_1 = \chi + \chi' + 2\chi''$, $k_2 = \lambda_\chi + \lambda'_\chi + 2\lambda''_\chi$, $k_3 = \eta_\chi + \eta'_\chi + 2\eta''_\chi$, and $k_4 = 2(\gamma + \gamma')$. The $k_5 \dots k'''_5$ are rather complicated polynomials of $\Gamma_\chi, \Gamma'_\chi$, and Γ_χ in Eq.(A5), such that $k_5 = k_5(\Gamma_\chi, \Gamma'_\chi)$, $k'_5 = k'_5(\Gamma_\chi, \Gamma''_\chi)$, $k''_5 = k''_5(\Gamma_\chi)$, and $k'''_5 = k'''_5(\Gamma'_\chi)$. Their details are not necessary for the following examination of the minimal condition. The first derivatives of the full potential given in Eq.(A2) are

$$\begin{aligned} \frac{\partial V}{\partial v} &= 2v \{ K(m_H^2, u^2, v_i^2, w_i^2) + \eta v^2 \} + 2\kappa (3v_2^2 v_1 - v_1^3) + k_4 u (v_1 w_1 + v_2 w_2) \\ \frac{\partial V}{\partial u} &= 2u \{ K(m_h^2, v^2, v_i^2, w_i^2) + \lambda_h u^2 \} + 2\kappa_s (3w_2^2 w_1 - w_1^3) + k_4 v (v_1 w_1 + v_2 w_2) \\ \frac{\partial V}{\partial v_1} &= 2v_1 \{ K(m_\varphi^2, u^2, v^2, w_i^2) + \Lambda_a (v_1^2 + v_2^2) \} + 6\kappa v (v_2^2 - v_1^2) + k_4 uv w_1 + k_5 v_2 w_1 w_2 \\ \frac{\partial V}{\partial v_2} &= 2v_2 \{ K(m_\varphi^2, u^2, v^2, w_i^2) + \Lambda_a (v_1^2 + v_2^2) \} + 12\kappa v v_1 v_2 + k_4 uv w_2 + k_5 v_1 w_1 w_2 \\ \frac{\partial V}{\partial w_1} &= 2w_1 \{ K(m_s^2, u^2, v^2, v_i^2) + \Lambda_s (w_1^2 + w_2^2) \} + 6\kappa_s u (w_2^2 - w_1^2) + k_4 uv v_1 + k_5 v_1 v_2 w_2 \\ \frac{\partial V}{\partial w_2} &= 2w_2 \{ K(m_s^2, u^2, v^2, v_i^2) + \Lambda_s (w_1^2 + w_2^2) \} + 12\kappa_s u w_1 w_2 + k_4 uv v_2 + k_5 v_1 v_2 w_1, \end{aligned} \quad (\text{A10})$$

where each K denotes the part that corresponds to the coefficients of linear terms. The vevs, $v_1 \neq 0$ and $v_2 = 0$, can make the potential minimum, when the following conditions are satisfied.

$$\begin{aligned} \left(\frac{\partial V}{\partial v_1} \right)_{v_2=0} &= 2v_1 (K + \Lambda_a v_1^2) - 6\kappa v v_1^2 + k_4 uv w_1 = 0 \\ \left(\frac{\partial V}{\partial v_2} \right)_{v_2=0} &= k_4 uv w_2 + k_5 v_1 w_1 w_2 = 0 \end{aligned} \quad (\text{A11})$$

$$\begin{aligned}
\left(\frac{\partial^2 V}{\partial v_1 \partial v_2}\right)_{v_2=0} &= k_5 w_1 w_2 > 0 \\
\left(\frac{\partial^2 V}{\partial w_1 \partial w_2}\right)_{v_2=0} &= 4\Lambda_s w_1 w_2 + 12\kappa_s u w_2 > 0.
\end{aligned}
\tag{A12}$$

It is clear that any of w_1 and w_2 should not be zero to satisfy the above conditions. According to the symmetry of the potential under the interchange of σ_1 and σ_2 , vevs can be taken as $w_1 = w_2 = w$. Thus, in summary, the following vevs of the fields in Eq.(A1) can be adopted for the masses of leptons:

$$\begin{aligned}
\langle H \rangle &= v \\
\langle \Phi \rangle &= (v_1, 0) \\
\langle h \rangle &= u \\
\langle \Sigma \rangle &= (w, w).
\end{aligned}
\tag{A13}$$

Then, the derivatives in Eqs.(A10) reduces to the following conditions:

$$\begin{aligned}
2v\{m_H^2 + k_1 u^2 + 2k_3 w^2 + \eta v^2\} - 2\kappa v_1^3 + k_4 u v_1 w &= 0 \\
2u\{m_h^2 + k_1 v^2 + 2k_2 v_1^2 + \lambda_h u^2\} + 4\kappa_s w^2 + k_4 v v_1 w &= 0 \\
2v_1\{m_\varphi^2 + \Lambda_a v_1^2 + \Lambda_b v^2 + k_2 u^2 + (2k'_5 + k'''_5)w^2\} - 6\kappa v v_1^2 + k_4 u v w &= 0 \\
2w\{2m_s^2 + 4\Lambda_s w^2 + 2\Lambda_c u^2 + 2k_3 v^2 + (2k'_5 + k'''_5)v_1^2\} + k_4 u v v_1 &= 0
\end{aligned}
\tag{A14}$$

According to Eq.(A11) and Eq.(A12), $k_4 < 0$ and $k_5 > 0$ are necessary. The mass matrices are examined upon the assumptions of $0.0001 < u/w < 5$ and $v/v_1 < 1$ with weak hierarchy. The assumptions do not show any conflicts with either the minimum conditions in Eq.(A14) or the phenomenological constraints, $\mathcal{O}(m_h^2, m_\varphi^2, m_s^2) > m_H^2$, since the constraints on masses and vevs contain a sufficient number of independent parameters.

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- [1] K. Abe *et al.* [T2K Collaboration], Phys. Rev. Lett. **107**, 041801 (2011)
 - [2] P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. Lett. **107**, 181802 (2011)
 - [3] Y. Abe *et al.* [DOUBLE-CHOOZ Collaboration], Phys. Rev. Lett. **108**, 131801 (2012)
 - [4] F. P. An *et al.* [DAYA-BAY Collaboration], Phys. Rev. Lett. **108**, 171803 (2012)
 - [5] J. K. Ahn *et al.* [RENO Collaboration], Phys. Rev. Lett. **108**, 191802 (2012)

- [6] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).
- [7] M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rept. **460**, 1 (2008)
- [8] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002)
- [9] P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003)
- [10] A. Zee, Phys. Lett. B **630**, 58 (2005)
- E. Ma, Phys. Rev. D **72**, 037301 (2005)
- K. S. Babu and X. G. He, arXiv:hep-ph/0507217.
- E. Ma, Mod. Phys. Lett. A **20**, 2601 (2005)
- G. Altarelli and F. Feruglio, Nucl. Phys. B **741**, 215 (2006)
- X. G. He, Y. Y. Keum and R. R. Volkas, JHEP **0604**, 039 (2006)
- X. G. He and A. Zee, Phys. Lett. B **645**, 427 (2007)
- R. R. Volkas, arXiv:hep-ph/0612296.
- F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B **775**, 120 (2007) [Erratum-ibid. **836**, 127 (2010)]
- W. Grimus, L. Lavoura and P. O. Ludl, J. Phys. G **36**, 115007 (2009)
- T. Araki, J. Mei and Z. z. Xing, Phys. Lett. B **695**, 165 (2011)
- [11] N. Haba, A. Watanabe and K. Yoshioka, Phys. Rev. Lett. **97**, 041601 (2006)
- [12] C. S. Lam, Phys. Rev. Lett. **101**, 121602 (2008)
- [13] C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP **0606**, 042 (2006)
- [14] H. Zhang, Phys. Lett. B **655**, 132 (2007)
- [15] F. Bazzocchi, L. Merlo and S. Morisi, Nucl. Phys. B **816**, 204 (2009)
- F. Bazzocchi, L. Merlo and S. Morisi, Phys. Rev. D **80**, 053003 (2009)
- [16] R. Z. Yang and H. Zhang, Phys. Lett. B **700**, 316 (2011)
- [17] S. Morisi and E. Peinado, Phys. Lett. B **701**, 451 (2011)
- [18] N. W. Park, K. H. Nam and K. Siyeon, Phys. Rev. D **83**, 056013 (2011)
- [19] S. Morisi, K. M. Patel and E. Peinado, Phys. Rev. D **84**, 053002 (2011)
- [20] D. Meloni, S. Morisi and E. Peinado, J. Phys. G **38**, 015003 (2011)
- [21] P. V. Dong, H. N. Long, C. H. Nam and V. V. Vien, Phys. Rev. D **85**, 053001 (2012)
- [22] X. Chu, M. Dhen and T. Hambye, JHEP **1111**, 106 (2011)
- [23] E. Ma, arXiv:hep-ph/0409075.
- [24] S. L. Chen, M. Frigerio and E. Ma, Phys. Rev. D **70**, 073008 (2004) [Erratum-ibid. D **70**,

079905 (2004)]

[25] J. Kubo, H. Okada and F. Sakamaki, Phys. Rev. D **70**, 036007 (2004)